Virtual Aristotelean Physics

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Abstract

Aristotle's physics provides a coherent set of physical laws which predict physical events less well than Newton's, but fit human expectations better. Embedded in a virtual reality, they thus provide a system which the user can more quickly learn to control, being more intuitive than Newton's laws and more consistent than *ad hoc* movement rules. Being simpler and cheaper to implement, they also give smoother behaviour (for given hardware investment) than the usual classical mechanics.

1 Introduction

'Correct' physics is often too CPU-costly to implement in a VR scheme intended for building layout, medical imaging, *etc.*; rather than build Newton's laws into the system, it is common to seek cheaper algorithms. Currently this is usually done by *ad hoc* kludging of the physics. By rolling science back five hundred years, we find a consistent, economical scheme in which objects can (for instance) be thrown more easily to a desired position than in the real world. In for instance a realistically scaled office environment there are usually points which are 'out of reach'. If the user is not to be for ever navigating the point-of-view around, throwing becomes a vital replacement for the dragging which has become a standard 2D positioning tool.

Sometimes, virtual reality must simulate 'objective' reality as closely as possible. For example, a virtual golf club or tennis racquet in a sports-training system, or an aircraft in a flight simulator, should closely model the original behaviour.

This behaviour is (a) complex and (b) hard to control. These features (desirable in a sport) conflict with the needs of such virtual reality applications as 3D design and layout systems. The virtual world used for these should be user-friendly; intuitive, easily learned and easily controlled. Moreover (since many details must be endlessly updated) the simpler it is numerically, the smoother its behaviour on given hardware can be.

Newtonian physics is not intuitive. It can seem so, to people thoroughly trained in it and struggling to move on to quantum mechanics, but it took the genius of Galileo and Newton to

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see past the *apparent* behaviour of matter and reach the 'simple' laws we now call Classical Mechanics. For two millenia Aristotle's physics[1] had satisfied those who thought about matter, because it was a systematization of that apparent behaviour—and a good one. It is intuitive, and is particularly good for VR, being economical to model. We illustrate this here with three examples: the way that an object moving in a straight line slows to a stop, the way a straight-thrown object slows and falls, and the idea of a 'curve ball'. These can be applied (for instance) to the motion of objects that leave the hand, either sliding in a straight line across the floor or thrown through the air.

2 Impetus

At the centre of Aristotle's physics is the fact that moving things tend to stop.

In modeling the Solar System, which is full of moving things which do *not* stop, this principle needed crystal 'wheels within wheels' to transmit the push that kept the planets moving: it made the Earth itself need pushing in Copernicus's system, which few could therefore accept. Galileo's perception that a steadily moving object *experiences* no force, and Newton's development of this to the law that force is required to *change* straight-line motion rather than to sustain it, led to the first great quantitative triumph of physics.

The earth-bound successes of classical physics came later, because friction complicates Newton immensely. A calculus student can early solve the (linearized) pendulum equation $\ddot{x} = -x$; the functions $\cos t$ and $\sin t$ are easily checked to be solutions. When a friction term is added, we move into Ordinary Differential Equation class, with all the subtleties of heavy versus light damping—does x cross zero once at most, or again and again?—even for linear friction, which is unrealistically simple. (For starters, 'static friction' is greater than the friction at low non-zero speed, so frictional force is not even continuous as a function of $\dot{x}$.) If the forces are smooth functions of position x and velocity $\dot{x}$, a moving object can never perfectly stop; for example the lightly damped linear oscillator has vibrating solutions $e^{-kt}\cos t$ and $e^{-kt}\sin t$ which decay exponentially with a 'half-life' of $\ln k/2$; the amplitude halves, and halves, and halves again, but never reaches exact zero. (Heavily damped slowing is similar, but let us not describe here exactly what decays exactly exponentially.)

'In practise', of course, a clock does run down (when the vibrations are too small to perceive, when they are lost in thermal fluctuations, when they are smaller than the 10^{-17} meters some physicists regard as the shortest possible length...) This adds another layer of complexity to the description, and numerically puts a logical test into the iteration: when some number has fallen below some fixed ε, we free the resources that were computing that motion, and regard that object as 'stopped'.

In contrast, take the Aristotelean view, that motion continues until impetus is exhausted. Aristotle was not very quantitative, but we may take impetus as proportional to velocity and implement something like this:

$$\begin{aligned} \mathsf{velocity\_step} &:= \mathsf{[some\ constant]} \\ \mathsf{step\_number} &:= \mathsf{initial\_velocity/velocity\_step} \\ \mathsf{do\ step\_number\ times} & \end{aligned} \tag{2.1}$$

```
{
velocity  :=  velocity − velocity_step
position  :=  position + velocity
display position
}
```

which involves no multiplies inside the loop—for a time step of 1 time-unit—and no explicit logical test. (The test of whether a 'number of times' has been reached is usually highly optimized by a compiler.) The object moves gracefully to a well-defined stop at the easily computed time T = **step_number** (Fig. 1).

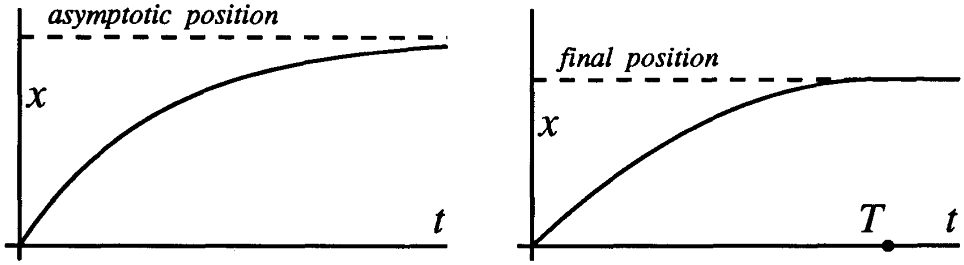

Figure 1. Exponential versus Aristotelean slowing, in one space dimension.

3 Ballistics

The same concept of 'impetus' applies to the Aristotelean theory of ballistics, with the added idea that processes happen *in sequence.* Galileo worked out the now-standard description of the parabolic path of a cannon ball replacing the Aristotelean theory (Fig. 2, top right) that the ball would go straight until its impetus was exhausted, and then fall[1]. (He constructed gunnery tables, and a protractor/plumb line combination for gunners to measure gun elevation, dividing a right angle into eight 'points': blank, 1, 2,.... 'Point blank range' was 'zero elevation necessary'.)

The pre-Galilean textbooks did not look unreasonable from the gunner's viewpoint (Fig. 2, bottom). The angle of an object in the sky is not easily judged, and the change of angle with time is not strikingly different. The contrast would be greater to a UN observer from beside the battle (Fig. 2, top), but such observers were rare.

From the VR viewpoint, Aristotelean ballistics are easily modeled. First model the ascent just as in (2.1), except that 'velocity' is now a vector, with x and y components. Second, model the fall as a *uniform* descent, with constant steps downward until the ground or target

[1]The modern textbook version is *not* intuitively obvious. An East European colleague told us of his military service, when a regular officer was explaining the ballistic parabola to a group of University students. One mischievously asked whether laying the gun on its side would also lay the parabola flat, so that the gun could shoot round corners. After some thought, the officer said "Theoretically, yes, but in practise it would be too difficult to handle the gun."

is reached. Aristotelean falling down or floating upwards is based on the principle that 'objects seek their own level', so there is no increase in impetus as they descend; they simply work away at moving to where they belong. Again, the approximation to real objects is different from that in high-school dynamics, but not necessarily worse; the Galilean[2] laws $v = v_0 - gt$ and $y = y_0 + v_t - gt^2/2$ ignore air resistance. Any object has a *terminal velocity* at which the downward force of gravity exactly balances the upward forces of buoyancy and fluid friction, so an object can descend at a steady rate—or steadily ascend, if buoyancy is enough. If Galileo had really performed the mythical Tower of Pisa experiment, and used a big cannon ball and a goat's dropping, he would *not* have found that all objects fall at the same speed. For objects whose weight is not large compared to their air resistance, steady fall $y = y_0 - v_{\text{limiting}}t$ is a better approximation than $y = y_0 + v_t - gt^2/2$, and is clearly cheaper in CPU time. Moreover, by implementing 'decay of impetus' first, and 'fall' only after the impetus is exhausted, we need less CPU time between frames within either stage than for Galilean calculations of both x and y between each visible step.

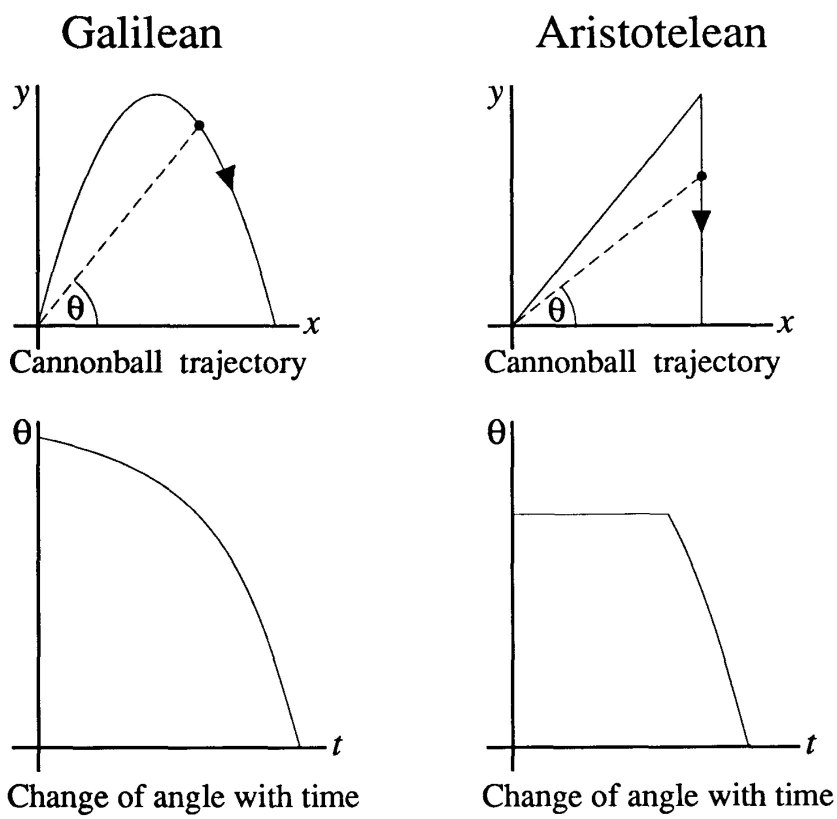

Figure 2. Aristotelean versus Galilean ballistics; views from the side and from the gun.

[2]As codified by Newton and given standard notation by Euler.

4 Circular Impetus

In one difference between Aristotle and Newton, Newton does have greater simplicity; his idea that straight line motion continues steady and straight, unless disturbed by a force. Newton's moons and planets move in circles and ellipses only because there is a continuing gravitational force acting on them, and at this point the mathematics does become more complicated.

The Aristotelean view regarded circles as 'perfect', so that celestial objects naturally moved in circles. This provided a simpler 'explanation' of the sky, except that—particularly when Earth is assumed as the center—the planets visibly do *not* move in circles, unless you mount new circular wheels on their rims, smaller circles on those,... and put planets on the final rims. (The first approximation improved when Copernicus suggested putting the centers near [not at] the Sun, but his final fit to the data involved even more extra circles than the geocentric model, which survived until Kepler's ellipses combined a sun-centered view with real computational improvement.)

The belief that circular motion persists as circular, however, remains psychologically attractive. Small boys far more often reinvent the curved gun barrel for firing in circles, as a way to shoot around corners, than the gun on its side for firing in horizontal parabolæ. (The 'parabola on its side' above was cognitive dissonance, applying naïve intuition to textbook physics: the circle is intellectually coherent pre-Galilean thinking.) The young bowler or pitcher has to learn, painfully, that the ball will not inherit curved motion from the hand; curving away from the simple path depends on the aerodynamics of a spun ball, a complicated transfer of angular momentum from rotation of the ball about its center to a turning on a larger scale. Few master this in practise (batsmen fear those who do), and the Sports Physics articles explaining it are hard even to read, let alone implement cheaply in VR. A virtual sports trainer for standard reality, of course, would have to implement this intricate physics; but for self-contained VR, why not implement the Aristotelean curve ball? Once again it is cheaper, and easier for a naïve user to learn. (The naïve user would also enjoy luring the skilled non-virtual player into a setting where carefully-learned counterintuitive reflexes no longer work!) The mathematics needed is as follows.

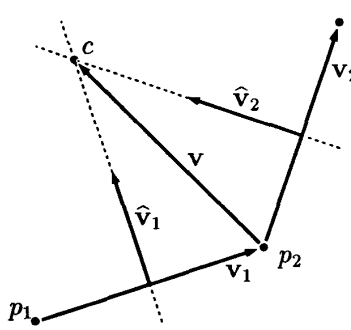

Figure 3. The common center of two rotation steps.

Suppose given three points $p_1 = (x_1, y_1, z_1)$, $p_2 = (x_2, y_2, z_2)$ and $p_3 = (x_3, y_3, z_3)$, the last three positions before the hand lets go (perhaps smoothed using previous points). We

seek a circular motion they fit. Define vectors $\mathbf{v}_1 = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$ and $\mathbf{v}_2 = (x_3 - x_2, y_3 - y_2, z_3 - z_2)$, and also $\boldsymbol{\omega} = \mathbf{v}_1 \times \mathbf{v}_2$, orthogonal to their common plane P. Set

$$\hat{\mathbf{v}}_1 = \boldsymbol{\omega} \times \mathbf{v}_1\,, \qquad \hat{\mathbf{v}}_2 = \boldsymbol{\omega} \times \mathbf{v}_2\,, \tag{4.2}$$

in P, so that (Fig. 3) we may write the vector $\mathbf{v}$ from (x_2, y_2, z_2) to the center of rotation as

$$\mathbf{v} = s\,\hat{\mathbf{v}}_1 - \tfrac{1}{2}\mathbf{v}_1 = t\,\hat{\mathbf{v}}_2 + \tfrac{1}{2}\mathbf{v}_2 \tag{4.3}$$

for some scalars s and t. Taking the cross-product with $\hat{\mathbf{v}}_2$ gives

$$s\,\hat{\mathbf{v}}_1 \times \hat{\mathbf{v}}_2 = \tfrac{1}{2}(\mathbf{v}_1 + \mathbf{v}_2) \times \hat{\mathbf{v}}_2\,, \tag{4.4}$$

where both sides are multiples of $\boldsymbol{\omega}$ since they are cross-products of vectors in $P = \boldsymbol{\omega}^{\perp}$. We can thus compute s either as the quotient

$$\frac{\big((\mathbf{v}_1 + \mathbf{v}_2) \times \hat{\mathbf{v}}_2\big) \bullet \boldsymbol{\omega}}{2\,(\hat{\mathbf{v}}_2 \times \hat{\mathbf{v}}_2) \bullet \boldsymbol{\omega}} \tag{4.5}$$

or more cheaply as the ratio of any non-zero component (x, y or z) in the two vectors $(\mathbf{v}_1 + \mathbf{v}_2) \times \hat{\mathbf{v}}_2$ and $2\,(\hat{\mathbf{v}}_2 \times \hat{\mathbf{v}}_2)$. Then the center of rotation $c = (c_x, c_y, c_z)$ is simply $c = p_2 + s\,\hat{\mathbf{v}}_1 - \tfrac{1}{2}\mathbf{v}_1$. A rotation by θ about this point, with axis in the $\boldsymbol{\omega}$-direction, may be written as

$$p \quad \mapsto \quad c + \cos\theta\,(p - c) + \sin\theta\,(p - c) \times \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} \quad \approx \quad p + \theta\,(p - c) \times \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} \tag{4.6}$$

to first order in θ; the small rotation about c which carries p_1 to p_2 (and p_2 to p_3 if $|\mathbf{v}_2| = |\mathbf{v}_1|$) is thus approximately

$$p \quad \mapsto \quad \mathsf{R}_\sigma(p) = p + \sigma\,(p - c) \times \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|}\,, \tag{4.7}$$

where $\sigma = |(p_1 - c) \times \mathbf{v}_1| \,/\, |\boldsymbol{\omega}|(p_1 - c) \bullet (p_1 - c)$ to match the angle $\theta = \angle p_1 c p_2$. In matrix terms,

$$\mathsf{R}_\sigma \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} + \sigma \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{bmatrix} x - c_x \\ y - c_y \\ z - c_z \end{bmatrix}, \tag{4.8}$$

a mere seven multiplies. Calling the pair $(\sigma, \boldsymbol{\omega})$ the *circular impetus*, we can apply

```
do step_number times
        {
                σ  :=  σ − impetus_step                    (4.9)
         position  :=  R_σ (position)
         display position
        }
```

Strictly, the aproximation in (4.6) means that iterating (4.7) will spiral outward from the rotation axis, instead of following a closed circle; but this effect is already minor if the steps $\mathbf{v}_1$ and $\mathbf{v}_2$ are small enough for the user to perceive smooth motion, and becomes weaker as σ shrinks, so that as a source of error it is smaller than the uncertainties in locating the user's hand. Effectively we have a circular motion, fitting the curvature and speed with which the object is released, and slowing to a smooth stop. No expensive evaluations of cosine, sine or exponentials are required.

5 Conclusions

We have described cheap, consistent schemes for the behavior of a thrown object, with and without gravity and 'curve ball' effects, for easy control of object position in a VR application.

Preliminary tests of our implementations show them to be both CPU cost-effective and user-friendly. It is clear that there are further intuitive, economic features of Aristotle's universe which can profitably be built in to a coherent, intuitive VR world. We will continue to explore these, and invite the contributions of those who know this universe better. It would also be interesting to investigate the VR potential of other well-developed systems of pre-Galilean mechanics, such as those of India and China. Such systematic choices could usefully be added to 'world-choosing' software such as the Artificial Reality Kit[3].

There is clearly some psychological demand on the user, in switching between standard and Aristotelean reality: but the latter is less of a wrench to the untrained intuition (the intuition without tuition!) than is Newton's, and far less so than objective mechanics in which lightspeed or Planck's constant play noticeable rôles—potentially enlightening, but very CPU-costly, alternate virtual realities.

References

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[3] Randall B. Smith, Experiences with the Alternate Reality Kit: An Example of the Tension between Literalism and Magic. Proceedings of ACM CHI+GI '87 Conference on Human Factors in Computing Systems and Graphics Interface, 1987.